

EIT phenomenon for the three-level hydrogen atoms and its application to the era of the cosmological recombination

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Abstract

The electromagnetically induced transparency (EIT) phenomenon in earlier universe is considered. We evaluated the elementary processes of the single scattering of photon on the hydrogen atom with the purpose of their use in the tasks of the radiation transfer theory. The additional function f , which depends on external conditions, is found. This function can be considered as an adjustment of the optical depth that leads to the necessity of modernization of the escape probability $p_{ij}(\tau_S) \rightarrow p_{ij}(\tau_S(1+f))$. The numerical values of f for the different schemes of atom in three-level approximation are given. It is found that the magnitude of function f could influence significantly on the formation of CMB in some partial cases.

1 Introduction

Electromagnetically induced transparency (EIT) is a well-known quantum interference phenomenon that arises when coherent optical fields couple to the states of a quantum system [1]. Interference effects arise, because in quantum mechanics the probability amplitudes must be summed and squared to obtain the total transition probability between the relevant quantum states. Interference effects can lead to profound modification of the optical and nonlinear optical properties of medium. In EIT the interference occurs between alternative transition pathways, driven by radiation fields. Interfering transition pathways can be induced by application of resonant laser fields to multilevel atomic systems. The most interesting example for this type of interference is the cancellation of absorption for a probe field, tuned in resonance to an atomic transition. EIT exhibits a laser-induced interference effect in the atom, which leads to an almost transparent medium.

In this paper we investigate the medium transparency constructed from hydrogen atoms under the cosmological recombination conditions. The transparency of primordial medium was achieved in cosmological recombination era, when the universe was cooled and the electrons combined with the protons, forming hydrogen atoms. After "recombination", photons were able to travel through the universe relatively unimpeded, and formed comprise the primordial background radiation that should be affect on generated hydrogen.

The problem of line transfer in medium can be described quite accurately with the Sobolev approximation [2]. Within the resonance the photon may be scattered in medium by the resonant line many times before it escapes from the matter. To account for resonant trapping in the Sobolev approximation, one should replace the normal spontaneous transition rate by the effective transition rate, modernized by the Sobolev escape probability. Theory of radiation transfer with the use of the Sobolev escape probability p_{ij} , where j is the upper level and i is the lower level, for the multilevel atoms have been described in [3].

The "standard" recombination calculation refers to the calculation with the "effective three-level hydrogen atom" that is widely used and first derived in [4], [5], where the "effective three-level hydrogen atom" means a hydrogen atom that includes the ground state, first excited state and continuum. In [3] it was noted that effective three-level hydrogen atom, which would completely neglect all levels above $n = 2$, would be a hopeless approximation. Good accuracy is obtained by considering an n -level atom, where n is large enough. In practice, authors [3] found that a 300-level atom is more than adequate for the description of the microwave background.

In this paper the other kind of three-level atomic system which includes only the bound states is considered. We describe this system as a zero approximation to demonstrate the possibility of the EIT in recombination era of earlier universe. The three-level hydrogen is chosen as a system, that can be adequately reduced to three levels when interaction with the pertinent electromagnetic fields are considered. The atomic selection rules require that two pairs of levels are dipole-coupled, while the transition between the third pair is dipole forbidden. This formulation allows to separate the resonant (one-photon) and non-resonant (two-, three-photon and etc) radiation escape processes based on different effects. As such a system, we consider a hydrogen atom with the ground and two excited states with the principal quantum number $n \leq 3$.

In Fig. 1 the basic three-level systems are shown. All of the level schemes, discussed in this paper, can be reduced to one or other of these schemes. We label the levels $|1 \rangle$, $|2 \rangle$, and $|3 \rangle$. The dipole-allowed transitions are between

states $|1\rangle$ and $|3\rangle$ and between states $|2\rangle$ and $|3\rangle$. Classification of the schemes then depends upon the relative energies of the three states: Fig. 1 a) ladder (or cascade) scheme with $E_1 < E_3 < E_2$, b) lambda (Λ) scheme with $E_1 < E_2 < E_3$, and c) vee (V) scheme with $E_3 < E_1$ and $E_3 < E_2$. In a lambda or ladder scheme state $|1\rangle$ is normally the ground state of the atom. The lambda scheme has a special importance due to the possible metastability of state $|2\rangle$.

Recently, on the basis of [6]-[8], the ladder scheme with $|1\rangle \equiv |1s\rangle$, $|3\rangle \equiv |2p\rangle$ and $|2\rangle \equiv |3s\rangle$ under cosmological conditions of cosmic microwave background (CMB) was considered [9]. It was found that the additional function f is arisen in optical depth τ_S . The maximal value of the f was found and is about 1.5% in case of exact resonances (when the both detunings are equal to zero). In case of exact two-photon resonance, when frequencies of fields are close but differ slightly to the corresponding resonances and the total detuning is equal to zero, we found $f \approx 0.95\%$. It was expected that these modifications should bring a strong impact to the determination of the key cosmological parameters [10].

In present paper we describe three-level systems: ladder system with fine structure splitting, vee scheme and Λ scheme for the hydrogen atom. We study the response of the three-level system on the external fields originating from photons emitted during the recombination period and evaluate the absorption coefficient, which can be applied to the radiation transfer theory. The physics of atom interaction with photon fields can be understood through interfering pathways description which corresponds to multi-photon process defined by power series expansion over fields amplitudes.

2 Theory and basic equations

New cosmological experiments on the CMB anisotropy investigations allow to know about processes in earlier universe in details and to understand the physical processes, playing crucial role on extra large scales in universe. Detailed understanding of the recombination process is important for modeling the power spectrum of CMB anisotropies. CMB distribution can be obtained on the basis of the radiation transfer theory, which was strongly re-examined recently, see, for example, [3], [11]-[17]. We will rely on the work [3] for the search of possible application of the EIT phenomena to astrophysics. In accordance to [9] we omit the evaluation of the radiation transfer equations and restrict ourselves by the derivation of the additional terms to the "standard" absorption coefficient, which is included in the optical depth.

The Sobolev escape probability can be defined as

$$p_{ij} = p_{ij}(\tau_S), \quad (1)$$

where j is the upper level and i is the lower level, τ_S is the Sobolev optical depth that can be presented, in the special case, in the form $\tau_S = \frac{\lambda_{ij}\tilde{k}}{|\nu'|}$. Here \tilde{k} is the integrated line absorption coefficient, ν' is the velocity gradient which is given by the Hubble expansion rate $H(z)$ and λ_{ij} is the central line wavelength. The monochromatic absorption coefficient or opacity is $k = \tilde{k}\phi(\nu_{ij})$, ν_{ij} is the frequency for a given line transition and $\phi(\nu_{ij})$ is the normalized line profile.

Absorption coefficient depends strongly on the external conditions and requires the particular consideration for the each case. In presence of an external field the opacity can be defined from the density matrix ρ_{ij} as follows:

$$k = \frac{Nd_{ij}^2\omega_{ij}}{2\varepsilon_0\Omega_{ij}}\text{Im}\{\rho_{ij}\}, \quad (2)$$

where ε_0 is the vacuum permittivity, N is the number of atoms, d_{ij} is the dipole matrix element, Ω_{ij} is the Rabi frequency and ω_{ij} is the frequency of the corresponding transition $|j\rangle \rightarrow |i\rangle$.

Further we assume that during the recombination period in earlier universe the hydrogen atoms reach their ground states with emission of all the spectra of photons corresponding to the continuum-bound and bound-bound transitions in an atom. Consideration of the "atom-field" system can be restricted by the spontaneous emission rates. Collisional excitation and ionization can be omitted because at the relevant temperatures and densities they are negligible [3].

In this case the application of the density matrix formalism in three-level approximation seems to be more simple and useful. The clear description of the density matrix theory and its applications can be found, for example, in [18], [19]. As it was shown in [9] the line profile can be separated out in the imaginary part of density matrix in approximation of weak fields and it was established

$$\text{Im}\{\rho_{ij}\} \approx \tilde{\phi}(\nu_{ij}) \left(1 + f(\Omega_p^2, \Omega_c^2, \Delta_p, \Delta_c)\right), \quad (3)$$

where $\tilde{\phi}(\nu_{ij})$ is the line profile which is multiplied by the corresponding Rabi frequency. The $f(\Omega_p^2, \Omega_c^2, \Delta_p, \Delta_c)$ represents the dimensionless function which depends on the Ω_p , Ω_c and detunings Δ_p , Δ_c (which are corresponded to the "probe" and "coupling" fields, respectively).

Thus for the three-level system the integrated line absorption coefficient can be presented in the form:

$$\tilde{k}_{ij} = \frac{\pi d_{ij}^2 N \omega_{ij}}{4\epsilon_0} [1 + f(\Omega_p^2, \Omega_c^2, \Delta_p, \Delta_c)] . \quad (4)$$

The radiation transfer theory is based on the elementary processes of the electron scattering on the atom at rest. In fact, dependence of the escape probability, $p_{ij}(\tau_S)$, for the photon in the line wing due to the expanding universe can have the complex form. The Sobolev approximation works at a certain phase which is well known. In more complicated cases the diffusion approximation have to be applied [11]. However, we restrict ourselves by the task of taking into account the external fields in the single scattering of external photon. In more common case we can write

$$\tau_S \equiv \tau_S^0 [1 + f(\Omega_p^2, \Omega_c^2, \Delta_p, \Delta_c)] , \quad (5)$$

where the optical depth τ_S^0 corresponds to the "standard" definition. Then we should replace $p_{ij}(\tau_S)$ by the $p_{ij}(\tau_S^0 (1 + f(\Omega_p^2, \Omega_c^2, \Delta_p, \Delta_c)))$

Further we illustrate the importance and calculate the contribution of the function $f(\Omega_p^2, \Omega_c^2, \Delta_p, \Delta_c)$ for the different kind three-level systems, see Fig. 1.

3 Derivation of the imaginary part of the density matrix element

Consider the three-level system which is subjected to the two external coupling and probe fields. In order to describe the response of such system on a field that was formed during the recombination era, we apply the density matrix formalism. Taking in mind the continual spectrum of the cosmic microwave background, we can always pick up the resonant frequency even in such way that detuning will be vanish, and, thus, avoid consideration of the Doppler effect. Moreover, in equations we leave only spontaneous widths as a natural parameter of the spectral line due to the collisional excitation (relaxation) and ionization can be omitted at the relevant temperatures and densities [3].

The time evolution of the density matrix elements is given by the master equation (in atomic units):

$$\frac{d}{dt}\rho_{ij} = -i \langle i | [H, \rho] | j \rangle , \quad (6)$$

where $|i\rangle$ denotes the energy states of the atom, $i = 1, 2, 3$, H is the Hamiltonian of the system, $H = H_0 + H_{int}$, which can be presented in the rotating wave approximation like:

$$H_0 = \omega_1|1\rangle\langle 1| + \omega_2|2\rangle\langle 2| + \omega_3|3\rangle\langle 3| \quad (7)$$

$$H_{int} = -\frac{1}{2} [\Omega_p e^{-i\omega_p t}|3\rangle\langle 1| + \Omega_c e^{-i\omega_c t}|3\rangle\langle 2| + c.c.] ,$$

where Ω_p , Ω_c are the Rabi frequencies of the probe and coupling field, respectively, the detunings from resonance are given by Δ_p and Δ_c , ω_p and ω_c are the frequencies of the corresponding electric fields, ω_i with $i = 1, 2, 3$ are the energies of the atomic states. The Rabi frequency is defined by $\Omega = \mathbf{d}_{ij}\mathbf{E}$, where \mathbf{E} is the electric field strength for the corresponding transition. Finally, detunings can be defined as $\Delta_p = \omega_3 - \omega_1 - \omega_p$ and $\Delta_c = \omega_3 - \omega_2 - \omega_c$.

The optical response of a medium to a resonant light is determined by the off-diagonal elements ρ_{21} , ρ_{31} and ρ_{32} of the density matrix, for which $\rho_{ij} = \rho_{ji}^*$ holds. In the master equation approach the effect of decoherence is taken into account by including damping terms, where it is assumed that the off-diagonal matrix elements ρ_{ij} decay with the respective rates Γ_{ij} . In the case of ρ_{32} and ρ_{31} for the Λ scheme, this contains spontaneous emission from state $|3\rangle$ to $|1\rangle$ or $|2\rangle$, respectively, with the total spontaneous emission rate $\Gamma_3 = \Gamma_{31} + \Gamma_{32}$. In the presence of damping, the time evolution of the coherences is then given by the Lindblad equations [19]:

$$\dot{\rho}_{ij} = -i\omega_{ij}\rho_{ij} - i[H_{int}, \rho]_{ij} - \gamma_{ij}\rho_{ij}, \quad i \neq j \quad (8)$$

$$\dot{\rho}_{ii} = -i[H_{int}, \rho]_{ii} + \sum_{\omega_j > \omega_i} \Gamma_{ij}\rho_{jj} - \sum_{\omega_j < \omega_i} \Gamma_{ji}\rho_{ii}.$$

Here Γ_{ij} gives the rate per atom at which population decays from the level j to i , γ_{ij} in absence of collisional processes can be represented as

$$\gamma_{ij} = \frac{1}{2}(\Gamma_i + \Gamma_j), \quad (9)$$

where Γ_i denotes the total decay width of the level $|i\rangle$, i.e.

$$\Gamma_i = \sum_{k(\omega_k < \omega_i)} \Gamma_{ki}. \quad (10)$$

Thus for the three-level system, Fig. 1, the master equations for the density matrix element can be written in the form:

$$\begin{aligned}\dot{\rho}_{12} &= -(\gamma_{12} + i\omega_{12})\rho_{12} - i\Omega_p\rho_{32} + i\Omega_c\rho_{13}, \\ \dot{\rho}_{23} &= -(\gamma_{23} + i\omega_{23})\rho_{23} + i\Omega_c(\rho_{22} - \rho_{33}) + i\Omega_p\rho_{21}, \\ \dot{\rho}_{13} &= -(\gamma_{13} + i\omega_{13})\rho_{13} - i\Omega_p(\rho_{11} - \rho_{33}) + i\Omega_c\rho_{12},\end{aligned}\quad (11)$$

where $\omega_{ij} \equiv \omega_i - \omega_j$, Ω_p corresponds to the probe field with $|1\rangle \rightarrow |3\rangle$ transition and Ω_c corresponds to the coupling field with $|2\rangle \rightarrow |3\rangle$ transition.

Introducing new variables $\tilde{\rho}_{31} = \rho_{31}e^{i\omega_p t}$, $\tilde{\rho}_{32} = \rho_{32}e^{i\omega_c t}$ and $\tilde{\rho}_{21} = \rho_{21}e^{i(\omega_p + \omega_c)t}$ for the cascade scheme (rotating wave approximation) we recieve

$$\begin{aligned}\dot{\rho}_{12} &= -(\gamma_{12} - i(\Delta_p + \Delta_c))\rho_{12} - i\Omega_p\rho_{32} + i\Omega_c\rho_{13}, \\ \dot{\rho}_{23} &= -(\gamma_{23} + i\Delta_c)\rho_{23} + i\Omega_c(\rho_{22} - \rho_{33}) + i\Omega_p\rho_{21}, \\ \dot{\rho}_{13} &= -(\gamma_{13} - i\Delta_p)\rho_{13} + i\Omega_p(\rho_{11} - \rho_{33}) + i\Omega_c\rho_{12},\end{aligned}\quad (12)$$

The Eqs. (12) can be easily converted to the Λ and V schemes by the replacement $\Delta_c \rightarrow -\Delta_c$ and $\Delta_p \rightarrow -\Delta_p$, respectively.

Full population of the ground state corresponds to the condition:

$$\begin{aligned}\rho_{11}(0) &= 1 \\ \rho_{22}(0) &= \rho_{33}(0) = 0.\end{aligned}\quad (13)$$

Equations (12) can be solved in the adiabatic regime or steady-state approximation, i.e. $\dot{\rho}_{ij} = 0$ that assumes approximation of the weak fields (the populations of the states are changed weakly). Analyze of the solution of the Eqs. (12) for the different kind of schemes was given in a lot number of papers, see for example [1], [6]-[8], [18], [19]. The response of the atoms to the weak probe field resonant to the $|1\rangle \rightarrow |3\rangle$ transition is the quantity of interest for EIT.

For the ρ_{31} and ρ_{32} (Ξ scheme) matrix elements in the adiabatic regime we obtain:

$$\begin{aligned}\rho_{31} &= -\frac{i\Omega_p [(\gamma_{12} + i\delta)(\gamma_{32} + i\Delta_c) + \Omega_p^2]}{(\gamma_{13} + i\Delta_p) [(\gamma_{12} + i\delta)(\gamma_{32} + i\Delta_c) + \Omega_p^2] + (\gamma_{32} + i\Delta_c)\Omega_c^2}, \\ \rho_{23} &= -\frac{i\Omega_p^2\Omega_c}{(\gamma_{13} + i\Delta_p)\Omega_p^2 + (\gamma_{23} + i\Delta_c) [(\gamma_{12} + i\delta)(\gamma_{13} + i\Delta_p) + \Omega_c^2]}, \\ \rho_{21} &= -\frac{(\gamma_{23} + i\Delta_c)\Omega_p\Omega_c}{(\gamma_{13} + i\Delta_p)\Omega_p^2 + (\gamma_{23} + i\Delta_c) [(\gamma_{12} + i\delta)(\gamma_{13} + i\Delta_p) + \Omega_c^2]}\end{aligned}\quad (14)$$

where $\delta = \Delta_p + \Delta_c$.

Application of the EIT phenomena to the astrophysics on the basis of [6]-[8] was done in [9]. For short we will not illustrate the series expansion over Ω_p and Ω_c variables in assumption the smallness of the electric fields strength. Applying the same procedure like in [6]-[8] and separating out the Lorentz line profile the imaginary parts of the ρ_{31} can be written in the form, see [9]:

$$Im\{\rho_{31}\} \approx \frac{\gamma_{13}\Omega_p}{\Delta_p^2 + \gamma_{13}^2} [1 + f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)]. \quad (15)$$

The analytical form of the function f is too cumbersome and further we will operate with numerical values. In principal, expression (15) should be integrated over frequencies, but we omit these calculations (which we leave for the future works) and give only the estimates in assumption of the parametrical dependence of the function f on the detunings and Rabi frequencies. The matrix elements ρ_{23} and ρ_{21} are not given due to their suppression by the factor Ω_p , as it should be. This suppression corresponds to the fact that we consider the full population of the lowest (ground) atomic level.

4 Numerical evaluation

The corresponding physical meaning of the function f can be found in [9] and can be understood via the optical depth Eq. (5). The "standard" way of the optical depth definition can be done through the integrated line absorption coefficient that is the characteristic of medium. The optical depth (see Eqs. (31), (33) in [3]), originally, does not take into account the influence of the external field. However, from optics it is well known that external fields should change the parameters of medium, including the spectral characteristics. The graphs 2 show which processes are included in function f that appeared as a result of external field's influence. Namely, the a) graph corresponds to the "standard" definition of the Sobolev escape probability. However, the existence of the second (coupling) field leads to the delay of the electron on the excited states, impedes the final recombination, due to the additional processes depicted on the graphs b)-d). Function f is the dimensionless one and shows the branching ratio of the processes b)-d), i.e. the magnitude of this function reveals the contribution of the external fields in respect to the usual one-photon absorption process a).

While the universe further expanded and cooled down the electrons and protons tended towards formation of hydrogen atoms. The temperature at this era is

well known $T_e \approx 4500 - 3000$ K. After "recombination", the photons released were able to travel through the universe relatively undisturbed, and formed the primordial background radiation. Such photon environment (background) should have influence on the hydrogen atom. The field amplitudes for a circular polarized wave can be obtained from the (thermal-averaged) spectral energy density

$$\frac{c\varepsilon_0|E|^2}{4\pi} = \frac{2h\nu_{ij}^3\Delta\nu_{ij}}{c^2} \frac{1}{e^{\frac{h\nu_{ij}}{k_B T_e}} - 1}, \quad (16)$$

where c is the speed of light, k_B is the Boltzmann constant, h is Planck's constant and in further calculations we use $T_e = 3000$ K. The right-hand side of the equation above corresponds to the black-body distribution of the CMB, while left-hand side defines the (electrical) energy density. In order to avoid de-phasing problem we should choose $\Delta\nu_{ij} \sim \Gamma_i$. Thus, we can define the corresponding field amplitudes for the different Ξ , Λ and V schemes by the fixing of the states $|1\rangle$, $|2\rangle$ and $|3\rangle$.

4.1 Ladder or cascade scheme Ξ

In [9] the Ladder scheme with $|1\rangle = |1s\rangle$, $|2\rangle = |3s\rangle$ and $|3\rangle = |2p\rangle$ was considered. It was established that the maximal value of the f is about 1.5% in case of exact resonances (when both detunings are equal to zero). In case of exact two-photon resonance, when frequencies of fields are close but differ slightly to the corresponding resonances and the total detuning is equal to zero, we obtained $f \approx 0.95\%$. In this section we consider fine structure of the hydrogen levels. Due to the Lamb shift the $|2p\rangle$ state is lower than the $|2s\rangle$ and the corresponding levels we can set as: $|1\rangle \equiv |1s\rangle$, $|3\rangle \equiv |2p\rangle$ and $|2\rangle \equiv |2s\rangle$, Fig. 1 a). The level widths are equal to $\Gamma_{2s} = 8.229 \text{ s}^{-1}$, $\Gamma_{2p} = 6.26826 \cdot 10^8 \text{ s}^{-1}$ and the transition frequencies are $\nu_{31} = 2.4674 \cdot 10^{15}$ and $\nu_{23} = 1.057911 \cdot 10^9$ in s^{-1} . Hence for the spectral Ly_α line ν_{31} , setting $\Delta\nu_{ij} = \Gamma_{2p}$, we obtain

$$\begin{aligned} E_p &\approx 0.000068802 \text{ V/m} = 1.33799 \cdot 10^{-16} \text{ a.u.}, \\ E_c &\approx 0.0017496 \text{ V/m} = 3.40242 \cdot 10^{-15} \text{ a.u.}, \\ \Omega_p &\approx 9.96713 \cdot 10^{-17} \text{ a.u.}, \\ \Omega_c &\approx 1.02073 \cdot 10^{-14} \text{ a.u.} \end{aligned} \quad (17)$$

With the estimates Eqs. (17) the numerical results of the function f , see Eq. (15), for the different values of the detunings are presented in Table 1.

Table 1. The numerical values of the function $f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$ for the different magnitudes of detunings are presented. In the first column the values of the $f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$ are listed, in the second and third columns the detunings are written.

$f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$	$\Delta_p \text{ s}^{-1}$	$\Delta_c \text{ s}^{-1}$
$-1.38066 \cdot 10^{-4}$	0	0
$-1.38066 \cdot 10^{-4}$	$\Gamma_{2s} = 8.229$	$-\Gamma_{2s} = -8.229$
$8.28426 \cdot 10^{-5}$	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$-\Gamma_{2p} = -6.26826 \cdot 10^8$
$-8.12364 \cdot 10^{-6}$	$\Gamma_{2s} = 8.229$	$\Gamma_{2s} = 8.229$
$7.25115 \cdot 10^{-13}$	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$\Gamma_{2s} = 8.229$
$3.62557 \cdot 10^{-13}$	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$\Gamma_{2p} = 6.26826 \cdot 10^8$

From the Table 1 follows that the maximal value of the f is about 0.01% in case of the exact resonances (when the both detunings are equal to zero). In case of exact two-photon resonance, when frequencies of fields are close but differ slightly to the corresponding resonances and the total detuning is equal to zero, we found $f \approx 0.0083\%$. Thus, in contrast to the results [9], the maximum value of the function f for the ladder scheme lies beyond required accuracy of calculations of the formation processes of the cosmic microwave background. We should note also that in case of $\Delta\nu_{ij} = \Gamma_{2s}$ the maximal value of function f is of the order of 10^{-12} due to the smallness of the field amplitudes and we can conclude that EIT phenomena does not require the fine structure consideration for Ξ scheme.

4.2 Lambda scheme Λ

As it was mentioned above the system of equations (12) for the Λ scheme, see Fig. 1b), can be solved with the replacement $\Delta_c \rightarrow -\Delta_c$. Let us assume $|1\rangle \equiv |1s\rangle$, $|2\rangle \equiv |2s\rangle$, and $|3\rangle \equiv |3p\rangle$ then $\nu_{21} = 2.4674 \cdot 10^{15} \text{ Hz}$, $\nu_{31} = 2.9243 \cdot 10^{15} \text{ Hz}$, $\Gamma_2 \equiv \Gamma_{2s} = 8.229 \text{ s}^{-1}$ and $\Gamma_3 \equiv \Gamma_{3p} = 1.89802 \cdot 10^8 \text{ s}^{-1}$. Assuming also $\Delta\nu_{32} \approx \Gamma_{2s}$ and $\Delta\nu_{31} \approx \Gamma_{3p}$ in Eq. (16), we obtain

$$\begin{aligned}
E_p &\approx 2.45765 \cdot 10^{-18} \text{ a.u.}, \\
E_c &\approx 1.17805 \cdot 10^{-14} \text{ a.u.}, \\
\Omega_p &\approx 7.33142 \cdot 10^{-19} \text{ a.u.}, \\
\Omega_c &\approx 2.08452 \cdot 10^{-14} \text{ a.u.}
\end{aligned} \tag{18}$$

The numerical values of the function $f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$ are given in Table 2.

Table 2. The numerical values of the function $f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$ for the different magnitudes of detunings are presented. In the first column the values of the $f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$ are listed, in the second and third columns the detunings are written.

$f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$	$\Delta_p \text{ s}^{-1}$	$\Delta_c \text{ s}^{-1}$
-0.00189829	0	0
-0.00189829	$\Gamma_{2s} = 8.229$	$\Gamma_{2s} = 8.229$
0.00113955	$\Gamma_{3p} = 1.89802 \cdot 10^8$	$\Gamma_{3p} = 1.89802 \cdot 10^8$
-0.000112064	$\Gamma_{2s} = 8.229$	$-\Gamma_{2s} = -8.229$
$3.29832 \cdot 10^{-11}$	$\Gamma_{3p} = 1.89802 \cdot 10^8$	$\Gamma_{2s} = 8.229$
$1.64916 \cdot 10^{-11}$	$\Gamma_{3p} = 1.89802 \cdot 10^8$	$-\Gamma_{3p} = -1.89802 \cdot 10^8$

If we assume $\Delta\nu_{32} \approx \Gamma_{2s}$ and $\Delta\nu_{31} \approx \Gamma_{2s}$ then $\Omega_p \approx 1.52655 \cdot 10^{-22}$ and $\Omega_c \approx 2.08452 \cdot 10^{-14}$ in atomic units. The amplitude of the field E_c is the dominant and is equal to the field magnitude in previous case. Thus we will have the same results as in Table 2.

In case $\Delta\nu_{32} \approx \Gamma_{3p}$ and $\Delta\nu_{31} \approx \Gamma_{3p}$ we get

$$\begin{aligned}\Omega_p &\approx 7.33142 \cdot 10^{-19} \text{ a.u.}, \\ \Omega_c &\approx 1.00111 \cdot 10^{-10} \text{ a.u.}\end{aligned}\tag{19}$$

Estimates Eqs. (19) show that we can neglect by the terms which are higher order over Ω_p . However, the approximation Eq. (15) is correct under the special conditions due to the violation of the relation $\Omega_i/\gamma_{ij} \ll 1$, namely Ω_c/γ_{12} which is not a small parameter now. The solution ρ_{31} takes the form:

$$\rho_{31} \approx -\frac{i\Omega_p}{\gamma_{13} + i\Delta_p + \frac{\Omega_c^2}{\gamma_{12} + i\delta}}.\tag{20}$$

Taking into account the $\gamma_{12} = \Gamma_{2s}/2 \sim 10^{-16} \text{ a.u.}$, one can find that the behaviour of the matrix element ρ_{31} is not lorentzian for the vanishing detunings, $\Delta_p, \Delta_c \rightarrow 0$ or if $\Delta_p = \Delta_c$ ($\delta = 0$). In opposite case when one of the detunings is large enough approximation Eq. (14) is kept. The values of function f for some detunings are listed in Table 3.

Table 3. The notations are the same as in Tables 1, 2.

$f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$	$\Delta_p \text{ s}^{-1}$	$\Delta_c \text{ s}^{-1}$
0.000761156	$\Gamma_{3p} = 1.89802 \cdot 10^8$	$\Gamma_{2s} = 8.229$
0.000380479	$\Gamma_{3p} = 1.89802 \cdot 10^8$	$-\Gamma_{3p} = -1.89802 \cdot 10^8$
$-9.04486 \cdot 10^{-7}$	$\Gamma_{2s} = 8.229$	$\Gamma_{3p} = 1.89802 \cdot 10^8$

From the Tables 2 and 3 follows that the maximal value of the f is approximately 0.1% for the equal detunings, see Table 2, or is close to 0.1% for the estimates Eq. (20) in case of big Δ_p (first and second lines in Table 3).

Consider now fine structure of the Λ scheme. In this case we choose $|1\rangle \equiv |1s\rangle$, $|2\rangle \equiv |2s\rangle$ and $|3\rangle \equiv |2p_{3/2}\rangle$. Then $\nu_{31} \approx 2.46741 \cdot 10^{15}$, $\nu_{32} \approx 9.96903 \cdot 10^9$, $\Gamma_3 \equiv \Gamma_{2p} = 6.26826 \cdot 10^8 \text{ s}^{-1}$ and $\Gamma_2 \equiv \Gamma_{2s} = 8.229 \text{ s}^{-1}$. In order to estimate field amplitudes we apply again Eq. (16) with $\Delta_{31} \sim \Gamma_{2p}$ and $\Delta\nu_{32} \sim \Gamma_{2s}$:

$$\begin{aligned}\Omega_p &\approx 9.96632 \cdot 10^{-17} \text{ a.u.}, \\ \Omega_c &\approx 1.10204 \cdot 10^{-10} \text{ a.u.}\end{aligned}\tag{21}$$

Numerical results of the evaluation of the function f are listed in Table 4 which shows that at such estimates Eqs. (21) the contribution of the function f is negligible. When we define $\Delta_{31} \sim \Gamma_{2s}$ and $\Delta\nu_{32} \sim \Gamma_{2s}$ we got the analogous results due to the smallness of the field amplitudes.

Table 4. The notations are the same as in Tables 1, 2.

$f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$	$\Delta_p \text{ s}^{-1}$	$\Delta_c \text{ s}^{-1}$
$-1.60962 \cdot 10^{-10}$	0	0
$-1.60962 \cdot 10^{-10}$	$\Gamma_{2s} = 8.229$	$\Gamma_{2s} = 8.229$
$9.65771 \cdot 10^{-11}$	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$\Gamma_{2p} = 6.26826 \cdot 10^8$
$-9.46834 \cdot 10^{-12}$	$\Gamma_{2s} = 8.229$	$-\Gamma_{2s} = -8.229$
$8.45246 \cdot 10^{-19}$	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$\Gamma_{2s} = 8.229$

However, if we define $\Delta_{31} \sim \Gamma_{2p}$ and $\Delta\nu_{32} \sim \Gamma_{2p}$ we get

$$\begin{aligned}\Omega_p &\approx 9.96632 \cdot 10^{-17} \text{ a.u.}, \\ \Omega_c &\approx 9.61828 \cdot 10^{-14} \text{ a.u.}\end{aligned}\tag{22}$$

Table 5. The notations are the same as in Tables 1, 2.

$f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$	$\Delta_p \text{ s}^{-1}$	$\Delta_c \text{ s}^{-1}$
-0.0121124	0	0
-0.0121124	$\Gamma_{2s} = 8.229$	$\Gamma_{2s} = 8.229$
0.00729051	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$\Gamma_{2p} = 6.26826 \cdot 10^8$
-0.000729015	$\Gamma_{2s} = 8.229$	$-\Gamma_{2s} = -8.229$
$6.43847 \cdot 10^{-11}$	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$\Gamma_{2s} = 8.229$
$3.21924 \cdot 10^{-11}$	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$-\Gamma_{2p} = -6.26826 \cdot 10^8$

and consequent numerical results which are illustrated in Table 5.

Thus contribution of the $f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$ reaches the value about 1%. This effect is within the accuracy of the modern computations of CMB and, therefore, should be taken into account.

4.3 Vee scheme V

In this section we consider vee V scheme depicted in graph Fig. 1c). With the requirement the $|1\rangle \equiv |3p\rangle$, $|2\rangle \equiv |2p\rangle$ and $|3\rangle \equiv |1s\rangle$, we have $\nu_{23} = 2.4674 \cdot 10^{15} \text{ Hz}$, $\nu_{13} = 2.9243 \cdot 10^{15} \text{ Hz}$, $\Gamma_2 \equiv \Gamma_{2p} = 6.26826 \cdot 10^8 \text{ s}^{-1}$ and $\Gamma_1 \equiv \Gamma_{3p} = 1.89802 \cdot 10^8 \text{ s}^{-1}$. Assuming $\Delta\nu_{23} \approx \Gamma_{2p}$ and $\Delta\nu_{13} \approx \Gamma_{3p}$ in Eq. (16), we obtain

$$\begin{aligned}\Omega_p &\approx 7.33142 \cdot 10^{-19} \text{ a.u.}, \\ \Omega_c &\approx 9.96713 \cdot 10^{-17} \text{ a.u.},\end{aligned}\tag{23}$$

and numerical values of the function $f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$ are given in Table 6.

Finally, lets consider the fine structure for the V scheme, Fig. 1 c). We choose $|1\rangle \equiv |2p_{3/2}\rangle$, $|2\rangle \equiv |2p_{1/2}\rangle$ and $|3\rangle \equiv |1s\rangle$, Then we have $\nu_{23} = 2.4674 \cdot 10^{15} \text{ Hz}$, $\nu_{13} = 2.46741 \cdot 10^{15} \text{ Hz}$, $\Gamma_2 \equiv \Gamma_{2p_{1/2}} = 6.26826 \cdot 10^8 \text{ s}^{-1}$ and $\Gamma_1 \equiv \Gamma_{2p_{3/2}} = 6.26826 \cdot 10^8 \text{ s}^{-1}$. Assuming $\Delta\nu_{13} \approx \Gamma_{2p}$ and $\Delta\nu_{23} \approx \Gamma_{2p}$ in Eq. (16), we again obtain the vanishing values for the function $f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$ of the order 10^{-16} . Thus from Table 6 follows that such processes are unimportant and we can fully neglect by the consideration of V scheme.

Table 6. The notations are the same as in Tables 1, 2.

$f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$	$\Delta_p \text{ s}^{-1}$	$\Delta_c \text{ s}^{-1}$
$-5.70837 \cdot 10^{-16}$	0	0
$4.8248 \cdot 10^{-16}$	$\Gamma_{3p} = 1.89802 \cdot 10^8$	$\Gamma_{3p} = 1.89802 \cdot 10^8$
$4.52944 \cdot 10^{-16}$	$\Gamma_{3p} = 1.89802 \cdot 10^8$	$-\Gamma_{3p} = -1.89802 \cdot 10^8$
$4.14603 \cdot 10^{-16}$	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$\Gamma_{2p} = 6.26826 \cdot 10^8$
$-1.14167 \cdot 10^{-16}$	$\Gamma_{3p} = 1.89802 \cdot 10^8$	$\Gamma_{2p} = 6.26826 \cdot 10^8$
$1.76641 \cdot 10^{-17}$	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$-\Gamma_{2p} = -6.26826 \cdot 10^8$

4.4 Full settling in the $2s$ state

The $2s \leftrightarrow 1s$ two-photon decay process is able to substantially control the dynamics of cosmological hydrogen recombination [4], [5], allowing about 57% of all hydrogen atoms in the Universe to recombine through this channel. Taking into account this fact, we demonstrate briefly the influence of the EIT phenomena on the schemes Fig. 1. With the initial condition $\rho_{2s}(0) = 1$, but others are equal to zero, and estimates Eqs. (17) for the cascade scheme we receive numerical values listed in Table 7.

Table 7. The notations are the same as in Tables 1, 2.

$f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$	$\Delta_p \text{ s}^{-1}$	$\Delta_c \text{ s}^{-1}$
-0.000276132	0	0
-0.000276132	$\Gamma_{2s} = 8.229$	$\Gamma_{2s} = -8.229$
-0.0000552386	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$-\Gamma_{2p} = -6.26826 \cdot 10^8$
0.00003314	$-\Gamma_{2p} = -6.26826 \cdot 10^8$	$\Gamma_{2p} - \Gamma_{2s} = 6.26826 \cdot 10^8$
-0.0000162473	$\Gamma_{2s} = 8.229$	$\Gamma_{2s} = 8.229$
$7.25115 \cdot 10^{-13}$	$\Gamma_{2p} = 6.26826 \cdot 10^8$	$\Gamma_{2p} = 6.26826 \cdot 10^8$

For the Λ scheme for the matrix element ρ_{32} we get the negligible values of the function $f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)$, where the maximal value of the order 10^{-12} is reached at the zero detunings and Eqs. (18). Taking into account the fine structure with Eqs. (22) the maximal value of the function f was found on the level 10^{-8} . We should stress that for the case of the full population of the $2s$ state the matrix element ρ_{31} is suppressed by the magnitude of the factor $\Omega_p \Omega_c^2$ and, therefore, negligible.

5 Conclusions

In the last decade the research of the CMB anisotropy was carried out by many authors, see, for example, [11]-[17] where the hydrogen atom with the set of states about 300 levels was described. Such number of states is required in order to receive the accuracy about 0.1%. After recombination photons were able to travel through universe and formed the CMB. Generated by this way field should affect the primordial atoms. In this case we are confronted with the consideration of the phenomena of electromagnetically induced transparency which can lead to profound modification of the optical and nonlinear optical properties of medium. In this work we limited ourselves to the consideration of the EIT phenomena in respect to amendment of the medium characteristic, such as absorption coefficient, assuming its application to the radiation transfer theory that we leave for future works.

We employed the quantum optical evaluation of the integrated line absorption coefficient that assumes application of photon beams (laser). In astrophysics the photon beam diffusion is accounted for via the optical depth. Optical depth expresses the quantity of light removed from a beam by scattering or absorption during its path through a medium. The optical depth definition (see Eqs. (31), (33) in [3]), originally, does not take into account the influence of the external field. However, from optics it is well known that external fields should change the parameters of medium, including the spectral characteristics. Accordingly, we derived the absorption coefficient from the imaginary part of the density matrix element and the magnitudes of fields were deduced from the CMB distribution. We considered the different schemes in three-level approximation depicted on Fig. 1 for the hydrogen atom. The corresponding physical meaning of the function f Eq. (15) can be found, for example, in [9], see also [6]-[8].

The radiation transfer theory is based on the elementary processes of the electron scattering on the atom at rest. In fact, dependence of the escape probability, $p_{ij}(\tau_S)$, for the photon in the line wing due to the expanding universe can have the complex form. The Sobolev approximation works at a certain phase which is well known. In more complicated cases the diffusion approximation have to be applied [11]. However, we restricted ourselves the task of taking into account the external fields in the single scattering of external photon. We found there is the additional function f which depends on external conditions (field amplitudes and detunings from resonant frequencies). Introduction of the function f leads to the necessity of modernization of optical depth Eq. (5) and, therefore, $p_{ij}(\tau_S) \rightarrow p_{ij}(\tau_S (1 + f(\Omega_p, \Omega_c, \Delta_p, \Delta_c)))$. In principal, the expression Eq. (15)

should be integrated over frequencies. However, in this paper the problem was limited to the finding the maximal values of f . The values of function f for different schemes of the hydrogen atom in three-level approximation are listed in Tables 1-7.

The cascade scheme, Fig. 1 a), was considered in [9] and it was found that the maximal value of the f amounts to about 1.5% in case of exact resonances (when both detunings are equal to zero). In this paper, in assumption of the full population of the ground state, we found that the maximal value of the function f is achieved for the Λ scheme with the fine structure of levels, when the states are defined as $|1\rangle = |1s\rangle$, $|2\rangle = |2s\rangle$, $|3\rangle = |2p_{3/2}\rangle$ and field amplitudes are defined via $\Delta\nu_{31} = \Delta\nu_{32} = \Gamma_{2s}$. The maximal value of f is of the order 1.2%, see Table 5. The other significant example of the EIT phenomena is the case when $|1\rangle = |1s\rangle$, $|2\rangle = |2s\rangle$, $|3\rangle = |3p\rangle$ and $\Delta\nu_{32} = \Gamma_{2s}$, $\Delta\nu_{31} = \Gamma_{3p}$ with the estimates Eqs. (18). Under these conditions it is obtained that the order of the function f is about 0.2%, see Table 2. From Table 3 follows that the function f is close to the 0.1% for the vanishing detunings and estimates Eqs. (19). For all other cases we got the smaller values. However, on the basis of the work [9] and these results we can conclude that the modifications of the integrated line absorption coefficient, could influence significantly on the formation of CMB, Eq. (5).

Moreover, we examined the particular variants of the EIT phenomena manifestation with the full population of the $2s$ state which is caused by the fact that 57% of hydrogen atoms recombine through the $2s$ state [4]. However, it is established that in this case the EIT phenomenon is not essential at the modern level of the experimental accuracy. For example, consideration of the cascade scheme leads to the contribution about 0.03% (Table 7) and is more negligible in all other cases.

The problem of dephasing appears when defining the magnitude of the field. The effect of dephasing leads to the spectral line broadening. To prevent dephasing phenomena we confined the definition of the field by a narrow strip. In this case the width of the corresponding line appears as a natural parameter. In our calculations we used the following relation $\Delta\nu_{ij} = \Delta_p(\Delta_c)$. In addition we should note also the exponential behavior over temperature for the field amplitudes. With increasing of the temperature T_e larger values for the amplitudes could be obtained, see Eq. (16). Accordingly, the contribution of the EIT effect can be more significant for higher temperatures.

Acknowledgments

The work was supported by RFBR (grant No. 08-02-00026, No. 11-02-00168-a and No. 12-02-31010).

References

- [1] J.P. Marangos, T. Halfmann, Electromagnetically Induced Transparency. Chapter 14 in Handbook of Optics, Third Edition, vol. IV, Optical Properties of Materials, Nonlinear Optics, Quantum Optics, Editors: M. Bass, G. Li, E.V. Stryland, Mc Graw Hill, New York etc., 14.1-14.44 (2010).
- [2] V. V. Sobolev, Soviet Astr.-AJ **1**, 678 (1957)
- [3] S. Seager, D. D. Sasselov and D. Scott, Astrophys. J. Suppl. Series **128**, 407 (2000)
- [4] Ya. B. Zeldovich, V. G. Kurt, and R. A. Syunyaev, Zh. Eksp. Teor. Fiz. **55**, 278 (1968) [Sov. Phys. JETP Lett. **28**, 146 (1969)]
- [5] P. J. E. Peebles, Astrophys. J. **153**, 1 (1968)
- [6] R. M. Whitley and R. Stroud, Phys. Rev. A **14**, N. 4, p. 1498 (1976)
- [7] J. Gea-Banacloche, Y.-Q. Li, S.-Z. Jin and M. Xiao, Phys. Rev. A **51**, 576 (1995)
- [8] S. Wielandy and Alexander L. Gaeta, Phys. Rev. A **58**, 2500 (1998)
- [9] D. Solov'yev, V. Dubrovich and G. Plunien, J. Phys. B
- [10] A. Lewis, J. Weller and R. Battye, Mon. Not. R. Astron. Soc. **373**, 561570 (2006)
- [11] V. K. Dubrovich and S. I. Grachev, Astron. Lett. **31**, 359 (2006)
- [12] J. A. Rubiño-Martin, J. Chluba and R. A. Sunyaev, Mon. Not. R. Astr. Soc. **371**, 1939 (2006)
- [13] E. E. Kholupenko, A. V. Ivanchik, D. A. Varshalovich, Mon. Not. R. Astr. Soc. **378**, L39-L43 (2007)
- [14] J. Chluba and R. A. Sunyaev, Astronomy&Astrophysics **480**, 3, pp. 629-645 (2008)

- [15] C. M. Hirata, Phys. Rev. D **78**, 023001 (2008)
- [16] J. A. Rubiño-Martin, J. Chluba and R. A. Sunyaev, Astronomy&Astrophysics, **485**, 377 (2008)
- [17] J. Chluba and R. A. Sunyaev, Astronomy&Astrophysics, **512**, A53 (2010)
- [18] J. Weiner and P.-T. Ho, "Light-Matter Interaction: Fundamentals and Applications", Published by John Wiley&Sons, Inc., Hoboken, New Jersey 2003
- [19] R. W. Boyd, Nonlinear Optics, Third Edition, Academic Press, Orlando, (2008)

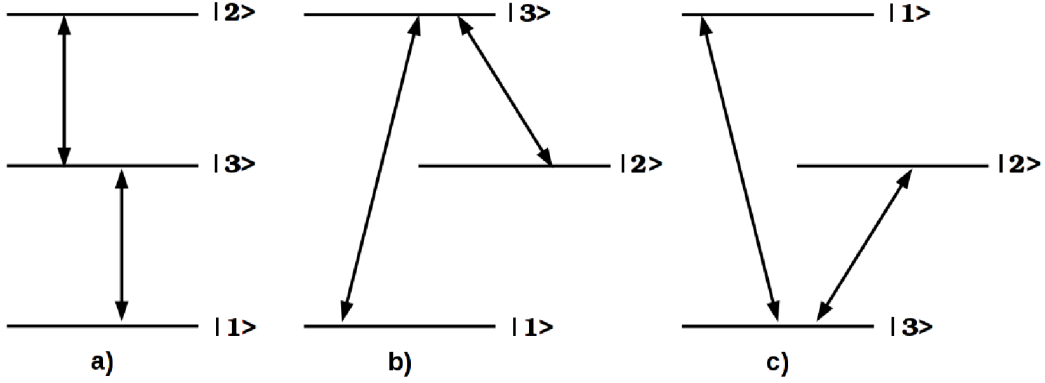


Figure 1: Basic three-level schemes: a) ladder or cascade (Ξ) scheme with $E_1 < E_3 < E_2$, b) lambda (Λ) scheme with $E_1 < E_2 < E_3$, c) vee (V) scheme with $E_3 < E_1$ and $E_3 < E_2$.

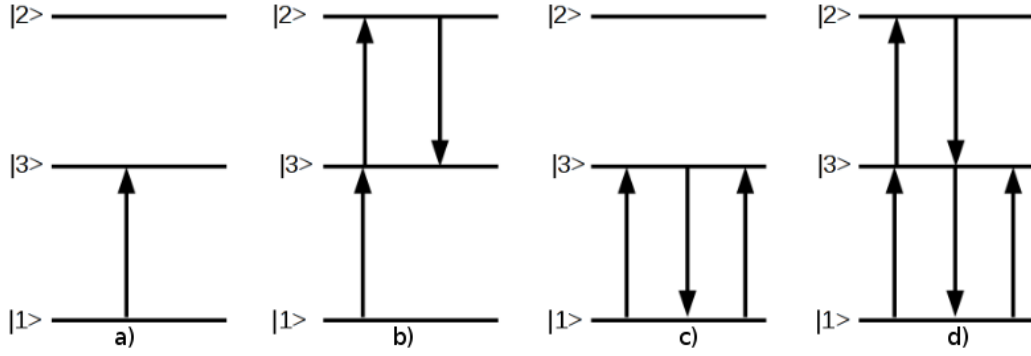


Figure 2: The transition processes, occurring in the three-level ladder system and corresponding to the different term of the imaginary part of Eqs. (15), are depicted: part a) of the figure represents the one-photon absorption processes which corresponds to the "standard" definition of the optical depth; part b), c) and d) represent the terms of the next orders in Ω_p and Ω_c appearing due to the presence of the external fields and derived via series expansion of the matrix elements Eqs. (14).